FRONTAL REGIMES IN THE DYNAMICS OF THE PHYSICAL ADSORPTION OF MIXTURES

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Analysis of the frontal regimes in the dynamics of physical adsorption of multicomponent mixtures for various types of theoretical models in which provision is made for a variety of interphase mass-exchange mechanisms is of theoretical and practical interest from the standpoint of choosing optimum regimes for adsorption separation of mixtures [1]. For the simplest theoretical model of the dynamics of adsorption (the equilibrium model of ideal expulsion), which is described by a narrowly defined hyperbolic system of quasilinear equations, the analysis of the frontal regime in the majority of cases [2, 3] is carried out with the aid of the Lax conditions [4-6]. For the theoretical models of the dynamics of mixture adsorption in which provision is made for the smearing factors, and these are described by a combined quasilinear system of equations, in [7] we have derived the conditions for the realization of the frontal regimes which depend not only on the form of the adsorption isotherms, but also, as in the case of the simplest model for hyperbolic equations, on the values of the mass-exchange coefficients characterizing the various mechanisms of interphase mass exchange. The hyperbolic systems of equations for adsorption dynamics in the case of mixtures may tolerate the existence of several (nonunique) frontal regimes within the framework of the Lax conditions [8].

In the present study we deal with the problem of selecting a single frontal regime through analysis of the analytical solutions for the equations of mixture adsorption dynamics, with provision for the approximate model equations which take into consideration the interphase mass exchange.

1. The frontal dynamics of the physical adsorption of multicomponent mixtures is described by a system of quasilinear equations for the material balance and the kinetics of the interphase mass exchange for each of the components in the mixture [7]:

$$\begin{gathered}\partial c_m/\partial t + u\,\partial c_m/\partial z + \delta\,\partial q_m/\partial t = \sum_{k=1}^{n} D_{mk}^{(l)}\partial^2 c_k/\partial z^2, \quad \delta = (1-\sigma)/\sigma, \\ \partial q_m/\partial t = \sum_{k=1}^{n} G_{mk}\,[f_k(c) - q_k], \quad 1 \leqslant m, \quad k \leqslant n. \end{gathered} \tag{1.1}$$

Here c_m is the concentration of the m-th component of the mixture in the moving phase; q_m is the concentration of the m-th component of the mixture in the adsorption phase; $f_m(c)$ represents the equations of the adsorption isotherms; u is the linear velocity of the flow; $D_{mk}^{(\ell)}$ is the coefficient of diffusion (m = k) and mutual diffusion (m ≠ k); G_{mk} is the coefficient of interphase mass exchange; n is the number of components in the mixture.

For the simplest equilibrium ($1/G_{mk} = 0$) model of an ideal expulsion ($D_{mk}^{(\ell)} = 0$), system (1.1) changes to the hyperbolic:

$$\begin{gathered}\partial c_m/\partial t + u\partial c_m/\partial z + \delta\partial f_m(c)/\partial t = 0, \\ 1 \leqslant m \leqslant n,\end{gathered}$$

which we write in matrix form

$$\begin{gathered}\partial c_m/\partial t + uB_{mk}\partial c_k/\partial z = 0, \\ B_{mk}^{-1} = E_{mk} + \delta f_{mk}, \quad f_{mk} = \partial f_m/\partial c_k\end{gathered} \tag{1.2}$$

(E_{mk} is the unit matrix). System of equations (1.2) for the initial and boundary conditions

$$c_m(0,\, t) = c_{0m},\ c_m(z,\, 0) = c^0{}_m,\ c_{0m},\ c^0{}_m = \text{const} \tag{1.3}$$

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permits the existence of two frontal regimes: a frontal regime of a type of concentrated traveling waves (shock waves)

$$c_m(z,t) = c_m(y),\ y = z - w_i t + y_{0i},\ y_{0i} = \text{const}, \tag{1.4}$$

and a frontal regime of diffusion-type concentrated waves (rarefaction waves)

$$c_m(z,t) = c_m(\xi),\ \xi = z/t. \tag{1.5}$$

For the adsorption isotherms $f_m(c)$ with monotonic eigenvalues μ_g the matrix f_{mk} of the condition for the existence (the Lax condition) of regimes such as (1.4) and (1.5), respectively, have the form [4, 6]:

$$\begin{gathered} v_i(c^{(i-2)}) > w_i > v_i(c^{(i-1)}),\quad c_{0m} > c_m^0, \\ c_m^{(i-2)} = c_m(y = -\infty),\quad c_m^{(i-1)} = c_m(y = +\infty),\quad 1 \leqslant i \leqslant n, \\ v_i(c^{(i-2)}) < v_i(c) < v_i(c^{(i-1)}),\quad c_m^{(i-2)} = c_m(\xi \to 0), \\ c_m^{(i-1)} = c_m(\xi \to \infty),\quad v_i = u/(1 + \delta\mu_{(n-i+1)}), \end{gathered} \tag{1.6}$$

where v_i are the eigenvalues of the matrix uB_{mk} [see (1.2)]. The velocity w_i is found from the Hugoniot equations [6]:

$$\begin{gathered} w_i = u/(1 + \delta[f_m(c)]/[c_m]), \\ [c_m] = c_m(y = -\infty) - c_m(y = +\infty), \\ [f_m(c)] = f_m(c(y = -\infty)) - f_m(c(y = +\infty)). \end{gathered} \tag{1.7}$$

According to (1.6), for the dynamics of mixture adsorption, system (1.2) may permit several frontal solutions such as (1.4) [8]. Therefore, the question of choosing a single physically realizable frontal regime of the (1.4) and (1.5) type within the scope of the Lax conditions remains open. For the solution of this problem let us examine a system of equations for the dynamics of mixture adsorption in which provision is made for the approximate equations which take into consideration interphase mass exchange:

$$\partial c_m/\partial t + u\,\partial c_m/\partial z + \delta\,\partial f_m(c)/\partial t = \sum_{k=1}^{n} \tau_{mk}\partial^2 c_k/\partial t^2, \tag{1.8}$$

which for isotherms of a specific form [see (1.9)] allow for an analytical solution. System (1.8) has been obtained out of (1.1) for $D_{mk}^{(\ell)} = 0$ and the approximate model equations for the kinetics of interphase mass exchange

$$q_m = f_m(c) - \sum_{k=1}^{n} \tau_{mk}\partial c_k/\partial t,\quad \tau_{mk} = \sum_{g=1}^{n} G_{mg}^{-1} f_{gk}.$$

From the physicochemical standpoint [1], the equations for the mixture adsorption isotherms $f_m(c)$ must satisfy the conditions $f_m(c) \geq 0$, $f_{mm} > 0$, $f_{mk} < 0$, $m \neq k$. For quasilinear mixture adsorption isotherms

$$f_m(c) = f_{0m} + \sum_{k=1}^{n} K_{mk}c_k,\quad K_{mm} > 0,\quad K_{mk} < 0,\quad m \neq k \tag{1.9}$$

and systems (1.1) and (1.8) it is possible to construct an analytical solution. For nonlinear isotherms of the form

$$f_m(c) = k_0 c_m + \sum_{g=1}^{n} b_{mg}c_m c_g,\quad k_0 > 0, \tag{1.10}$$

when

$$\tau_{mk} = \tau E_{mk},\ \tau > 0, \tag{1.11}$$

and systems (1.8), (1.11) for the nonsingular matrix b_{mk} an analytical solution can also be found.

The quasilinear system (1.1) and system of equations (1.8) may allow for a frontal regime of the diffusion type of concentrated waves for which

$$c_m(z, t) = c_m(\xi, t),\ \xi = z/t,\ 1 \leqslant m \leqslant n. \tag{1.12}$$

In the previous solution the quantity t is taken as the parameter. As $t \to \infty$ the solution of (1.12) tends asymptotically to the self-similar solution (1.5).

2. Let us examine the analytical solution of system (1.1), (1.8) for isotherms of the form of (1.9) and (1.10). The analytical solutions of system (1.1) for the quasilinear isotherms in (1.9) are found by means of the Laplace integral transform over time. After the transformations the asymptotic ($z, t \to \infty$) solution is written as follows:

$$\begin{gathered} c_m(z, t) \approx 1/2 \sum_{k=1}^{n} A_{mk} \operatorname{erfc}\left(y_k^{(i)}\right), \\ y_k^{(i)} = (z/w_i - t)/\left[2\left(b_k^{(i)} z\right)^{1/2}\right] \quad (1 \leqslant m, k, i \leqslant n), \\ \operatorname{erfc}(x) = 1 - \pi^{-1/2} \int_0^{\infty} \exp(-t^2)\, dt. \end{gathered} \tag{2.1}$$

To each wave index i ($1 \le i \le n$) we have corresponding eigenvalues of $b_k^{(i)}$ for the matrix $H_{gj}^{(i)} = \left(D_{mk}^{(l)}/w_i^2 + \delta \sum_{s=1}^{n} G_{gs}^{-1} f_{sj}\right)\Big/ u$ and the eigenvalues w_i of the matrix uB_{mk}, which we will number in the following manner:

$$\begin{gathered} w_1 < w_2 < w_3 < \ldots < w_n \ (1 \leqslant i \leqslant n), \\ w_i = u/(1 + \delta\mu_{(n-i+1)}) \end{gathered} \tag{2.2}$$

(μ_m are the eigenvalues of the matrix K_{gs}). It follows from the earlier solutions that the asymptotic solutions of (2.1) for (1.1) in the presence of (1.9) coincide with the asymptotic solutions of system (1.8), provided that the quantities $b_m^{(i)}$ equal the eigenvalues of the matrix τ_{mk}/u, and they coincide with (1.8), (1.11), provided that $\tau = u \max\limits_{i,m} b_m^{(i)}$. Strictly speaking, a frontal regime of the type of (1.4) occurs only for (1.2), while in the case of (1.1), according to (2.1), nonsteady regimes exist.

Let us take note of the specific unique features encountered in the frontal dynamics of adsorption for (1.9). In the case of nonlinear convex isotherms [for example, (1.10) with $b_{mm} < 0$) the strongly adsorbed component of the mixture [with the lowest value of b_{mm} in (1.9), for example b_{11}] in adsorption is "absorbed completely," since the concentration of the first component of the mixture for the frontal regime, moving at a velocity w_1, changes in the interval $c_{01} \ge c_1 \ge c_1{}^0$. The quantity w_1 in this case depends on the concentration of the mixture components [see (1.7)]. For (1.9) the strongly adsorbed component of the mixture (for example, the first) is independent of the mixture-component concentrations [see (2.2)], and in order to match (2.2) to (1.7) the concentration of the first component changes in the interval $c_{01} \le c_1 \le c_1{}^{(1)}$, $c_1{}^{(1)} > c_1{}^0$. The quantities $c_m{}^{(k)}$ are found from Eqs. (1.7), which are in agreement with (1.3).

Let us examine the solution of system of equations (1.8), (1.11) for the nonlinear isotherms (1.10). For the nonsingular matrix B_{mk}

$$c_m = \sum_{k=1}^{n} B_{mk} g_k \quad (1 \leqslant m,\ k \leqslant n) \tag{2.3}$$

the coefficients are chosen so that for each fixed value of m ($1 \le m \le n$) in $\sum\limits_{r,k=1}^{n} B_{mr}^{-1} b_{rk} B_{rj}$. $B_{km} g_j g_m$ only the diagonal terms of quadratic form will be different from zero, which will be the case when

$$g_m^2 b_m = \sum_{r,k=1}^{n} B_{mr}^{-1} b_{rk} B_{rm} B_{km} g_m^2 \neq 0,$$

$$\sum_{r,k=1}^{n} B_{mr}^{-1} b_{rk} B_{rj} B_{km} g_j g_m = 0 \quad (m \neq j,\ 1 \leqslant m,\ k,\ r,\ j \leqslant n).$$

In each case system (1.8), (1.11) "breaks down" into individual equations:

$$a_0 \partial g_m/\partial t + u \partial g_m/\partial z + \delta b_m \partial g_m{}^2/\partial t = \tau \partial^2 g_m/\partial t^2,$$
$$a_0 = 1 + \delta k_0,\ g_m(z, 0) = g_m{}^0 = \text{const},\ g_m(0, t) = h_m(t). \tag{2.4}$$

With substitution

$$g_m = g_m{}^0 + A_m \partial(\ln G_m)/\partial t,\ A_m = -\tau/(\delta b_m) \tag{2.5}$$

the nonlinear equation (2.4) reduces to the linear equation

$$a_0 \partial G_m/\partial t + u \partial G_m/\partial z = \tau \partial^2 G_m/\partial t^2\ (1 \leqslant m \leqslant n),$$
$$G_m(z, 0) = 0,\ G_m(0, t)/G^0 = \exp(b_m t),\ b_m = (g_{0m} - g_m{}^0)/A_m, \tag{2.6}$$
$$h_m(t) = \text{const};$$

$$G_m(0, t)/G^0 = \exp\left(\int_0^t (h_m(x) - g_m^0)\, dx\right), \quad h_m(x) = \text{var}. \tag{2.7}$$

For zero initial conditions when $h_m(t)$ = const the solution of Eqs. (2.6), with the aid of the Laplace integral transform with respect to the coordinate (s is the parameter of the integral transform), is obtained in the form

$$G_m(s, t) = G^0[\exp(b_m t) - \exp(\lambda t)]/(s - s_m),$$
$$s_m = b_m(b_m \tau - a_0)/u,$$
$$\lambda = a_0/(2\tau) - [a_0{}^2/(4\tau^2) + us/\tau]^{1/2},$$
$$G_m(z, t)/G^0 = \exp(b_m t + s_m z) - (1/2)\exp[a_0 t(1 - d_m)/(2\tau)]\,\text{erfc}\left(z_m^{(1)}\right) - \tag{2.8}$$
$$-(1/2)\exp[a_0 t(1 + d_m)/(2\tau)]\,\text{erfc}\left(z_m^{(2)}\right), \quad d_m = \left(1 + 4us_m/a_0^2\right)^{1/2},$$
$$z_m^{(1)} = (t - a_0 z d_m/u)/(4z\tau/u)^{1/2}, \quad z_m^{(2)} = (t + a_0 z d_m/u)/(4z\tau/u)^{1/2}.$$

When we take into consideration the expression for $G_m(z, t)$ and relationships (2.5), according to (2.3), we find the analytical solutions of system (1.8), (1.11) for nonlinear isotherms (1.10). By means of these we can describe the frontal regimes consisting of concentrated waves such as (1.4) and (1.12), for multicomponent mixtures in the presence of isotherms (1.10).

As an example, let us analyze the frontal regimes of a two-component mixture (n = 2) with the isotherms

$$f_1 = 4c_1 - c_1{}^2 - c_1 c_2,\ f_2 = 4c_2 - 0.9c_1 c_2 - 1.2c_2{}^2. \tag{2.9}$$

The hyperbolic system of equations (1.2) for isotherms (2.9) within the scope of conditions (1.6) allows for the existence of two frontal regimes (nonsingularity), consisting for each wave index i ($1 \leq i \leq 2$) of concentrated waves of the (1.4) and (1.5) type.

The distribution of concentrations along the z axis for a regime of the (1.5) type is found from the following system of ordinary differential equations:

$$dc_m/dc_1 = r_m^{(i)}(c)/r_1^{(i)}(c) \quad (2 \leqslant m \leqslant n), \quad \xi = v_i(c) \quad (1 \leqslant i \leqslant n), \tag{2.10}$$

derived from (1.2) and (1.5). Here $r_m{}^{(i)}$ are the proper eigenvectors of the matrix uB_{mk} for the eigenvalues of v_i ($1 \leq i \leq n$).

According to (1.6), (1.7), and (2.10), for isotherms (2.9) with $c_{01} = 1$, $c_1{}^0 = 0$, $c_{02} = 0.1$, $c_2{}^0 = 0$ we have

Regime I:

$$i = 1:\ v_1^- = u/(1 + 2.9463\delta), \quad v_1^+ = u/(1 + 3.0316\delta),$$
$$c_1^{(1)} = 1{,}093, \quad c_2^{(1)} = 0, \quad w_1 = u/(1 + 2.98\delta), \quad v_1^- > w_1 > v_1^+; \tag{2.11}$$

$$\begin{aligned}&i=2:\ v_2^-=u/(1+1.814\delta),\quad v_2^+=u/(1+4\delta),\quad w_2=u/(1+2.907\delta),\\&\qquad v_2^->w_2>v_2^+,\quad c_1^{(2)}=0;\end{aligned}$$

Regime II

$$\begin{aligned}&i=1:\ v_1^-=u/(1+2.9463\delta),\quad v_1^+=u/(1+2.8539\delta),\\&\qquad c_1^{(1)}=0,\quad c_2^{(1)}=1.146,\quad v_1^-<v_1(c)<v_1^+;\\&i=2:\ v_2^-=u/(1+1.25\delta),\quad v_2^+=u/(1+4\delta),\\&\qquad w_2=u/(1+2.625\delta),\quad v_2^->w_2>v_2^+,\quad c_2^{(2)}=0.\end{aligned}\tag{2.12}$$

The Lax conditions (1.6) for frontal regimes (2.11) and (2.12) are fulfilled and therefore for the indices i = 1 and 2 we achieve regime I in the form of traveling concentrated waves of the (1.4) type. For regime II, with i = 1, there exists an expanding concentrated wave of the type of (1.5), while with i = 2 we have a traveling concentrated wave similar to (1.4). Let us note that the eigenvalues of μ_m for the matrix f_{mk} in the case of isotherms (2.9) for a two-component mixture are quantities that are both real and different. The eigenvalues of $\mu_m(c_1, c_2)$ for isotherms (2.9) are monotonic functions of c_1, c_2. It has thus been demonstrated that for a two-component mixture (n = 2) and for monotonic eigenvalues of μ_m with $c_m{}^0 = 0$ $(1 \le m \le n)$ the hyperbolic system of quasilinear equations (1.2) allows the existence of several solutions. Let us note that when $c_m{}^0 \ne 0$ $(1 \le m \le n)$ the hyperbolic system (1.2) has a solitary solution consisting for each index i $(1 \le i \le n)$ in the presence of monotonic functions μ_m $(1 \le m \le n)$ of a combination of solutions such as for the concentrated waves (1.4) and (1.5). Indeed, with the aid of Eqs. (1.7) and (2.10) for each index i $(1 \le m \le n)$ we can find arcs [solutions $c_m = c_m(c_1)$ $(2 \le m \le n)$], which uniquely relate the points c_{0m}, $c_m{}^0$ $(1 \le m \le n)$.

In the case of the nonconservative hyperbolic system (1.2) for a mixture of three components (n = 3) the literature (in particular, see [6]) points up the possibility for the existence of several regimes consisting of self-similar solutions such as (1.4) and (1.5). The above-cited analysis for a two-component mixture (n = 2) provided the initial proof for the existence of several regimes in the case of a conservative hyperbolic quasilinear system of equations in the presence of mixture adsorption isotherms with monotonic eigenvalues of μ_m $(1 \le m \le 2)$.

Let us analyze the solutions of the quasilinear system (1.8), (1.11) for isotherms (1.10). The system allows the existence of type I and II frontal regimes, which when $h_m(t) = \text{const}$ are described by the solutions for (2.3) and (2.8), and with z, t → ∞ changing to the asymptotic, consist of a combination of concentrated waves such as (1.4) and (1.12). Specifying the constants g_{0m}, $g_m{}^0$ in (2.3) and (2.8), we can construct analytical solutions corresponding to regimes I and II, consisting in the case of z, t → ∞ of asymptotic solutions such as (1.4) and (1.12). Such an analysis shows the stability of nonidentical (two) solutions of the system of quasilinear equations (1.8) and (1.11), since as $\tau \to 0$ it tends monotonically to the solutions of (2.11) and (2.12).

Within the framework of the mathematical formulation of the problem dealing with the dynamics of mixture adsorption, the question dealing with the selection of a single regime remains open. In order to resolve this question it is essential that we use different concepts, in particular, physicochemical concepts. Based on the physicochemical concepts for the selection of a single physically realizable frontal regime in the dynamics of mixture adsorption $(c_{0m} > c_m{}^0)$ additional extremum conditions have been derived in [8]:

$$v_i=\min_{v,w}\left(\min_g v_i^{(g)};\ \min_s w_i^{(s)}\right)\quad(1\leqslant m\leqslant n),\tag{2.13}$$

according to which, for the frontal dynamics of adsorption, a regime with the lowest value for the velocity is physically attained. Conditions (2.13) have the following physicochemical sense: for the frontal dynamics of mixture adsorption it is the components of the mixture with the greatest value of adsorbability that are primarily absorbed, which corresponds to the largest eigenvalue of μ_g $(1 \le g \le n)$ or, according to (1.6), to the lowest velocity v_i $(1 \le i \le n)$.

For isotherms (2.9), according to the extremum condition (2.13), regime I is realized, since $w_1 = u/(1 + 2.98\delta) < v_1^- = u/(1 + 2.94\delta)$. These results are also confirmed by means of numerical experimentation, i.e., integration with a computer of the quasilinear system (1.8), (1.11), as well as confirmed by system (1.1), (1.2).

3. Let us examine the solution of system (1.8) for a matrix τ_{mk} of arbitrary form. Analysis of these solutions is of great applied interest for this matrix, since by means of such a system it becomes possible analytically for an entire series of important particular cases approximately to describe the processes of adsorption mixture separation in laboratory and industrial equipment, as well as to evaluate the accuracy of the approximate analytical formulas for the calculation of the parameters characterizing the processes of adsorption mixture separation.

In a number of cases, the experimental mixture adsorption isotherms can be described by approximate analytical expressions of the form

$$f_m(c) = k_m c_m + \sum_{g=1}^{n} b_{mg} c_m c_g \quad (1 \leqslant m,\ g \leqslant n). \tag{3.1}$$

For the nonsingular matrix τ_{mk} system (1.8), with consideration of the substitution

$$c_m = \sum_{k=1}^{n} A_{mk} g_k \quad (1 \leqslant m,\ k \leqslant n)\ , \tag{3.2}$$

changes into the following:

$$\partial g_m/\partial t + u\,\partial g_m/\partial z + \delta \sum_{s=1}^{n} A_{ms}^{-1}\,\partial f_s(c)/\partial t = \tau_m \partial^2 g_m/\partial t^2, \tag{3.3}$$

since the eigenvalues of the matrix τ_{mk} and $A_{mg}{}^{-1}\tau_{gj}A_{jk}$ coincide (τ_m are the eigenvalues of the matrix τ_{mk}). For isotherm (3.1) system (3.3) with consideration of the substitution

$$g_k = \sum_{p=1}^{n} L_{kp} s_p \quad (1 \leqslant k,\ p \leqslant n) \tag{3.4}$$

has the form

$$\begin{gathered} a_k\,\partial s_k/\partial t + u\,\partial s_k/\partial z + \delta \sum_{p,r=1}^{n} H_{kpr}\partial(s_p s_r)/\partial t = \tau_k^0 \partial^2 s_k/\partial t^2, \\ H_{kpr} = \sum_{i,m,j,l=1}^{n} L_{ki}^{-1} A_{im}^{-1} b_{mk} A_{mj} L_{jp} A_{kl} L_{lr}, \quad a_k = 1 + \delta k_k^0. \end{gathered} \tag{3.5}$$

Using

$$s_j = \sum_{l=1}^{n} P_{jl} r_l \quad (1 \leqslant j,\ l \leqslant n), \tag{3.6}$$

we write system (3.5) as

$$a_j \partial r_j/\partial t + u\,\partial r_j/\partial z + \delta b_j\,\partial r_j^2/\partial t = \tau_{0j}^0\,\partial^2 r_j/\partial t^2 \quad (1 \leqslant j \leqslant n),$$

if we select the coefficients of the matrices A_{mk}, L_{kp}, $P_{j\ell}$ so that

$$r_j^2 b_j = \sum_{k,p,r=1}^{n} P_{jk}^{-1} H_{kpr} P_{pj} P_{rj} r_j^2 \neq 0,$$

$$\sum_{k,p,r=1}^{n} P_{jk}^{-1} H_{kpr} P_{pj} P_{rs} r_j r_s = 0 \quad (j \neq s,\quad 1 \leqslant k,\ p,\ r,\ j \leqslant n).$$

In accordance with the above, the solution of the system of equations (1.8) for c_{0m}, $c_m{}^0$ = const in the case of the isotherms from (3.1) can be found on the basis of transformations (3.2), (3.4), and (3.6), with consideration of the solutions for (2.8) for various a_k, b_j, τ_m $(1 \leq k,\ j,\ m \leq n)$.

The analytical solutions obtained in the manner described here make it possible to analyze the entire multiplicity of frontal regimes in the multicomponent ($n \geq 3$) dynamics of adsorption ($c_{0m} > c_m^0$) and desorption ($c_{0m} < c_m^0$) for various values of the concentration c_{0m}, c_m^0 ($1 \leq m \leq n$).

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A SET OF STEADY-STATE SOLUTIONS OF THE EVOLUTION EQUATION FOR PERTURBATIONS IN ACTIVE-DISSIPATIVE MEDIA

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Steady-state periodic solutions have been calculated numerically. It is demonstrated that an even set of such solutions comes about as a consequence of a successive cascade of bifurcations.

In recent times, researchers into the wave processes in nonconservative media have expressed great interest in an equation of the form

$$H_t + 4HH_x + H_{xx} + H_{xxxx} = 0. \tag{1}$$

This interest is generated by the fact that in terms of form it is one of the simplest nonlinear equations which could possibly be imagined, so that with its appearance in the simulation of the nonlinear behavior of perturbations for a rather large class of active-dissipative media it functions for the latter in as extensive a role as the well-known KdV equation for conservative media.

Thus, in the description of the waves at the surface of a liquid film flowing freely down an inclined plane, such an equation has been derived in [1, 2], for the counterflow motion of a film and a gas we find the derivation of such an equation in [3], and for the perturbations at the boundaries separating two viscous liquids in a horizontal channel, the derivation of the equation is to be found in [4].

Linear stability analysis demonstrated that the trivial solution $H = 0$ of Eq. (1) is unstable relative to perturbations of the form $\exp[i\alpha(x - ct)]$ with wave numbers $\alpha < 1$ (perturbations with $\alpha > 1$ are attenuated). The growth of such perturbations over time can be curtailed through the action of nonlinear effects, as a result of which steady-state nonlinear regimes are formed.

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